Test Review: Periodic Functions

The test will be no calculator, so for the best practice, complete this review without the aid of a calculator or Desmos.

1. Select all that are solutions to the following equation.

$$3\tan^2\theta - 2\tan\theta - 1 = 0$$

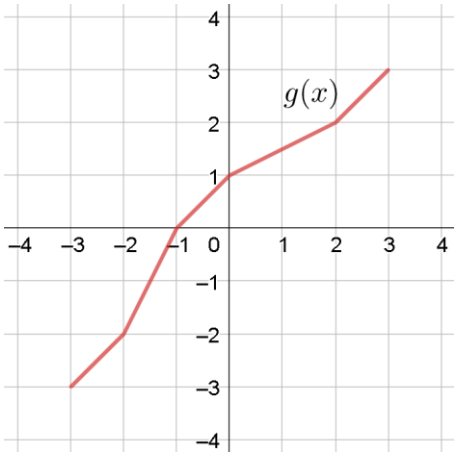
$\frac{\pi}{4}$	$-\arctan\left(\frac{1}{3}\right)$	$-\frac{\pi}{4}$	$\arctan\left(\frac{1}{3}\right)$
$\pi + \arctan\left(\frac{1}{3}\right)$	$2\pi + \arctan\left(\frac{1}{3}\right)$	$\frac{5\pi}{4}$	$\pi - \arctan\left(\frac{1}{3}\right)$

2. Select all that are solutions to the following equation.

$$3\sin^2\theta + 5\cos\theta - 1 = 0$$

$\arccos(2)$	$\pi - \arccos\left(\frac{1}{3}\right)$	$2\pi - \arccos\left(\frac{1}{3}\right)$	$2\pi + \arccos\left(\frac{1}{3}\right)$
$\pi - \arccos(2)$	$\arccos\left(\frac{1}{3}\right)$	$2\pi - \arccos(2)$	$\pi + \arccos\left(\frac{1}{3}\right)$

Use the below table of $f(x)$, graph of $g(x)$, and equation of $h(x)$ to answer #3-9

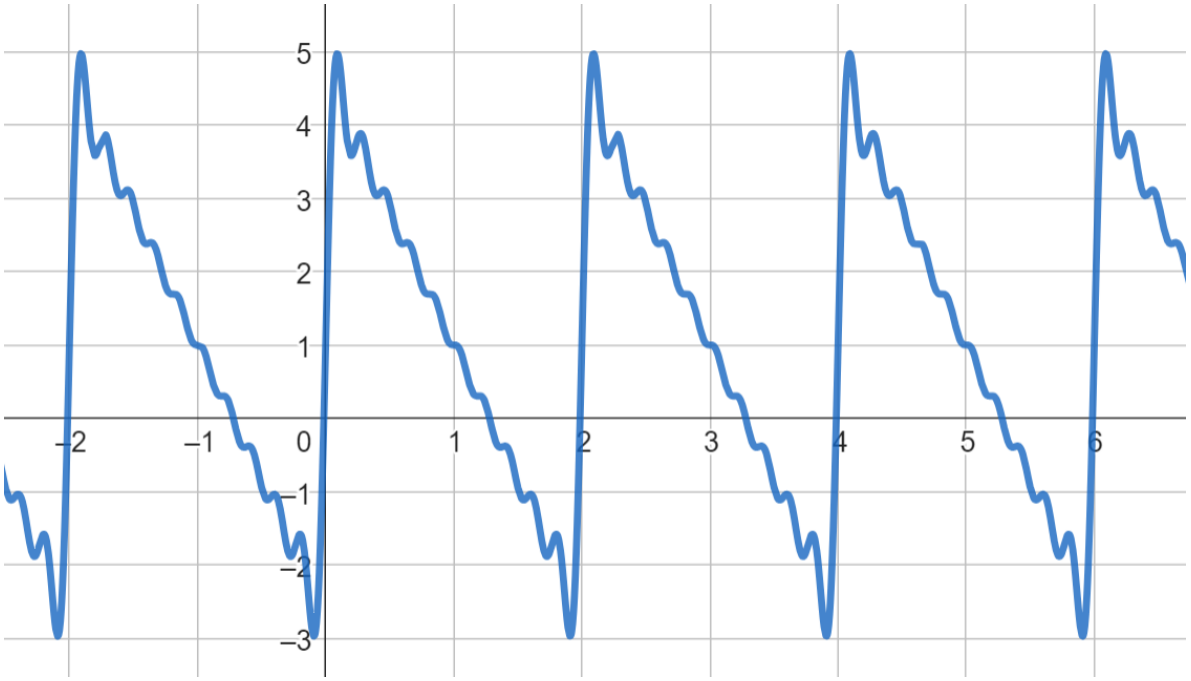


x	-3	-2	-1	0	1	2	3
$f(x)$	2	1	3	0	-3	-2	-1

$$h(x) = 3x - 6$$

3. Find $g^{-1}(1)$
4. Find $f^{-1}(-3)$
5. Find $h^{-1}(-2)$
6. Find $(f \circ g^{-1})(0)$
7. Find $(h^{-1} \circ f^{-1})(-2)$
8. Find $(h^{-1} \circ h^{-1})(12)$
9. Find an expression for $h^{-1}(x)$

Let $f(x)$ be the periodic function for which $y = f(x)$ is graphed below. Use this question to answer #10-13.



10. What is the period of $f(x)$?

11. What is the amplitude of $f(x)$?

12. Let $g(x) = 2f(x-1) + 3$.
What is the range of $g(x)$?

13. Which one of the following is NOT an odd function.
(A) $f(x) - 1$
(B) $f(x+1) - 1$
(C) $f(x-1) - 1$
(D) $f(x-1) + 1$

14. Let $g(x)$ be the function whose graph is the image of the graph of $f(x)$ after the following transformations in the following order: a vertical dilation by a factor of 2 followed by a horizontal translation by -1 units followed by a reflection over the y-axis.
   a. $g(x)$ may be expressed as $g(x) = af\big(b(x+c)\big)$. Find the values of a, b, and c.
   b. The point $(2,9)$ is on the graph of $f(x)$. Find the corresponding point on the graph of $g(x)$.
   c. Below are select values of the function $f(x)$. Find the value of $g(4)$.

x	-3	-2	-1	0	1	2	3	4
$f(x)$	-36	5	24	27	20	9	0	-1

Match each numbered description with the letter of the corresponding function.

15. The graph of $f(x)$ is reflected over the x-axis, then translated vertically by 2 units, then dilated vertically by a factor of 3

16. The graph of $f(x)$ is reflected over the x-axis, then dilated vertically by a factor of 3, then translated vertically by 2 units.

17. The graph of $f(x)$ is translated vertically by 2 units, dilated vertically be a factor of 3, then reflected over the x-axis.

18. The graph of $f(x)$ is dilated vertically be a factor of 3, then translated vertically by 2 units, then reflected over the x-axis.

(A) $-3f(x) - 2$

(B) $-3f(x) + 6$

(C) $-3f(x) - 6$

(D) $-3f(x) + 2$

19. Let $h(x) = x^2 - 2x$. The graph of which of the following functions is the image of $h(x)$ after a horizontal translation -1 unit.

(A) $x^2 - 4x + 2$

(B) $x^2 - 2x - 1$

(C) $x^2 - 1$

(D) $x^2 + 2$

20. As Zoey's heart beats, the volume of blood it contains increases and decreases periodically. The maximum volume of Zoey's heart is 350 mL while the minimum volume is 150 mL. It takes half a second to go from a time at which the heart is at maximum volume to a time at which the heart is at minimum volume. If $t = 0$ represents a time at which the volume of Zoey's heart is at a maximum, which of the following functions may be used to model the volume of her heart t seconds later?

(A) $250 + 100\cos(2\pi t)$

(B) $250 + 100\cos(\pi t)$

(C) $100 + 250\cos(2\pi t)$

(D) $100 + 250\cos(\pi t)$

21. The power consumed by my new freezer, in Watts, can be modelled by $P(t) = 12\cos\left(\frac{\pi}{2}t\right) + 18$ where t is the number of hours since I plugged it in. According to the model, which of the following is true?

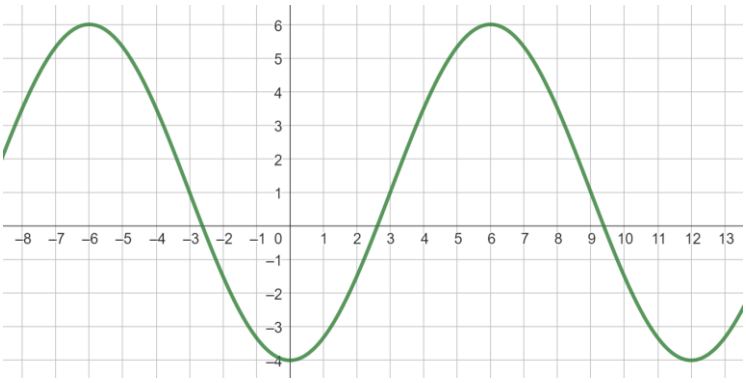
(A) The maximum amount of power the freezer uses is 12 W.

(B) Two hours after plugging in the freezer, it is using a maximum amount of power.

(C) The minimum amount of power the freezer uses is 6 W.

(D) There is a two hour interval between the times at which the freezer uses its minimum amount of power.

22. Which one of the following represents the function graphed to the right?



(A) $5\sin\left(\frac{\pi}{12}(x+3)\right)+1$

(B) $5\sin\left(\frac{\pi}{6}(x-3)\right)+1$

(C) $-5\sin\left(\frac{\pi}{12}(x-3)\right)+1$

(D) $-5\sin\left(\frac{\pi}{6}(x-3)\right)+1$

23. Find the range of $f(x)=4\sec(2x)+1$

24. Find the range of $g(x)=2\cos\left(x-\frac{\pi}{2}\right)-5$

25. Find the range of $h(x)=-6\tan\left(3\left(x-\frac{\pi}{4}\right)\right)$

26. Which one of the following describes the graph $y=3\cot\left(\frac{1}{3}\left(x+\frac{\pi}{4}\right)\right)-2$

(A) The spacing between the asymptotes on this graph is 6π. All points of inflection on this graph are on the line $y=2$.

(B) This is the graph of function that increases between consecutive asymptotes. All points of inflection on this graph are on the line $y=-2$.

(C) This is the graph of function that decreases between consecutive asymptotes. The spacing between the asymptotes is 3π.

(D) This is the graph of function that increases between consecutive asymptotes. The spacing between the asymptotes is 6π.

In questions #23-28, evaluate the expression.

27. $\arctan(-1)$

28. $\arcsin\left(-\frac{1}{2}\right)$

29. $\arccos(1)$

30. $\arctan\left(\frac{\sqrt{3}}{3}\right)$

31. $\arcsin(0)$

32. $\arccos\left(-\frac{\sqrt{2}}{2}\right)$

33. Consider the function $f(x)=\frac{1}{2}\csc\left(\frac{\pi}{3}(x+4)\right)$.

a. What is the period of this function?

b. What is the spacing between consecutive asymptotes on the graph of this function?

34. Which one of the following is an asymptote of $y = 8\sec\left(\frac{1}{4}x\right) + 2$

(A) $x = 0$ (C) $x = 2\pi$

(B) $x = \frac{\pi}{2}$ (D) $x = 3\pi$

35. Which one of the following is an asymptote of $y = 2\cot\left(\frac{\pi}{6}(x+5)\right)$

(A) $x = 0$ (C) $x = 2$

(B) $x = 1$ (D) $x = 4$

36. Solve over the interval $0 \le \theta < \frac{\pi}{2}$

$$8\sin^2\theta - 6\sin\theta + 1 = 0$$

(A) $\theta = \frac{\pi}{3}$ or $\theta = \arcsin\left(-\frac{1}{4}\right)$ (C) $\theta = \frac{\pi}{6}$ or $\theta = \arcsin\left(-\frac{1}{4}\right)$

(B) $\theta = \frac{\pi}{3}$ or $\theta = \arcsin\left(\frac{1}{4}\right)$ (D) $\theta = \frac{\pi}{6}$ or $\theta = \arcsin\left(\frac{1}{4}\right)$

37. Select all of the following that are solutions to the equation $\sin x = -\frac{2}{3}$

(A) $-\arcsin\left(\frac{2}{3}\right)$

(B) $\arcsin\left(\frac{2}{3}\right)$

(C) $\pi - \arcsin\left(\frac{2}{3}\right)$

(D) $\pi + \arcsin\left(\frac{2}{3}\right)$

(E) $2\pi - \arcsin\left(\frac{2}{3}\right)$

38. Let $f(x)$ be a periodic function with period 8 and amplitude 6. Let $g(x) = 4x$. Let $h(x) = 2x + 1$. Find the period and amplitude of $h\left(f\left(g(x)\right)\right)$.

39. What are all values of θ, $0 \le \theta \le 2\pi$ for which $2\cos\theta \ge 1$ and $2\sin\theta \le \sqrt{2}$?

(A) $\left[0, \frac{\pi}{4}\right] \cup \left[\frac{5\pi}{3}, 2\pi\right]$

(B) $\left[0, \frac{\pi}{3}\right] \cup \left[\frac{3\pi}{4}, \frac{5\pi}{3}\right]$

(C) $\left[\frac{\pi}{4}, \frac{\pi}{3}\right] \cup \left[\frac{3\pi}{4}, \frac{5\pi}{3}\right]$

(D) $\left[\frac{\pi}{4}, \frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3}, 2\pi\right]$

40. What are all values of θ, $-\pi \leq \theta \leq \pi$. for which $(\cos\theta - 1)(2\cos\theta + 1) < 0$?

41. Let n be the function given by $n(x) = 6\arccos(x^2) + \pi$. Find all input values in the domain of n that yield an output value of 3π.

42. Let p be the function given by $p(x) = 10\tan^{-1}(x - 1)$. Solve $p(x) = \frac{5\pi}{2}$ for all values of x in the domain of p.

43. The table below gives values of $f(x)$ and $g(x)$ at selected values of x. Functions f and g are defined for all real numbers.

x	0	1	2	3	4
$f(x)$	3	4	1	2	0
$g(x)$	2	1	0	4	3

a. Find $(f \circ g)(2)$ b. Find $(g \circ f)(0)$ c. Find $(f \circ f)(4)$

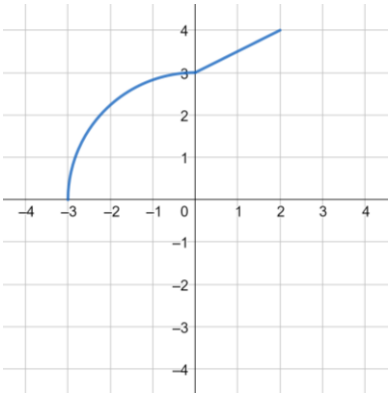
44. Let $f(x) = \frac{1}{x^2-4}$ and let $g(x) = \sqrt{x+3}$.
   a. Find an expression for $(f \circ g)(x)$

   b. Select all of the following that are in the domain of $(f \circ g)(x)$?

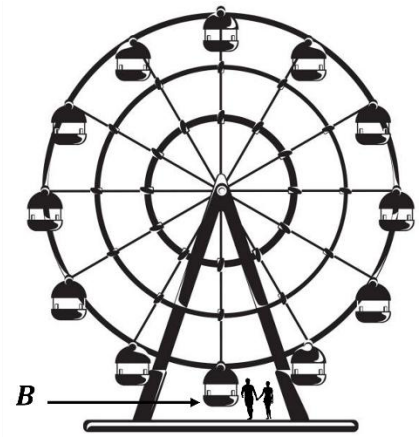
   (A) -4 (B) -3 (C) -2 (D) -1

   (E) 0 (F) 1 (G) 2 (H) 13

45. To the right is the graph of $r(x)$. Find the domain and range of $r^{-1}(x)$.



46. To the right is the graph of $r(x)$. Sketch the graph of $r^{-1}(x)$.

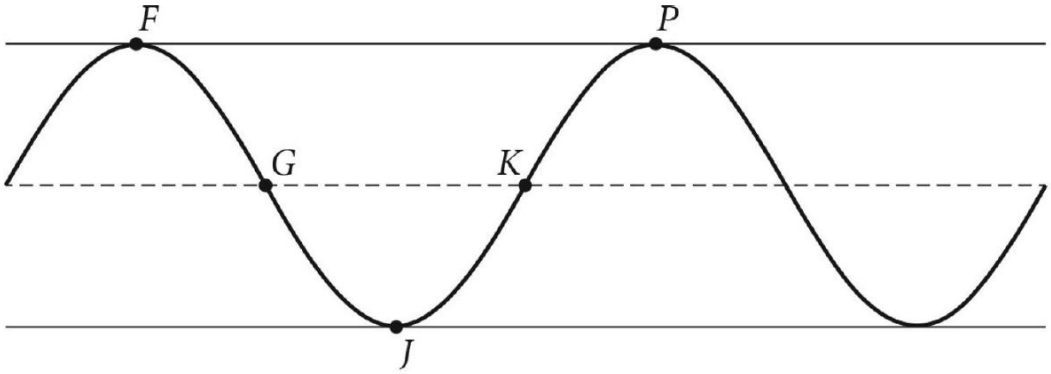


Note: Figure not drawn to scale.

47. The carts of a Ferris wheel are 40 feet from the center, rotate in a clockwise direction and complete 1 rotation every 180 seconds. At time $t = 50$, point B is where people enter the Ferris wheel, at the point of minimum height as indicated in the figure. At this time, point B is 5 feet above the ground. As the Ferris wheel goes around, the distance between Point B and the ground periodically increases and decreases.

    The sinusoidal function h models the distance between B and the ground, in meters, as a function of time t in seconds.

    (A) The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, I, J, and K are labeled on the graph. No scale is indicated, and no axes are presented.
    Determine possible coordinates $\big(t, h(t)\big)$ for the five points: F, G, I, J, and K.



    (B) The function h can be written in the form $h(t) = a\cos\big(b(t + c)\big) + d$. Find values of constants a, b, c, and d.

    (C) Refer to the graph of h in part (A). The t-coordinate of G is t_1 and the t-coordinate of J is t_2.

    (i) On the interval (t_1, t_2), which of the following is true about h?

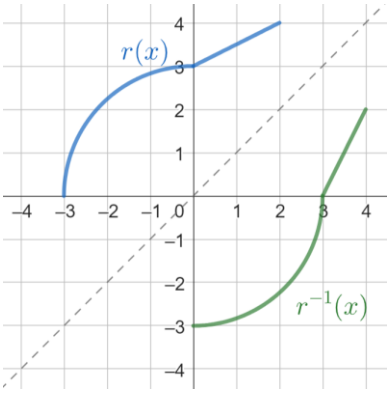
    a. h is positive and increasing
    b. h is positive and decreasing
    c. h is negative and increasing
    d. h is negative and decreasing

    (ii) On the interval (t_1, t_2), describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.

Answers to Periodic Functions Test Review

1. $-\arctan\left(\frac{1}{3}\right),\ \frac{\pi}{4}, \pi-\arctan\left(\frac{1}{3}\right), \frac{5\pi}{4}$
2. $\pi-\arccos\left(\frac{1}{3}\right), \pi+\arccos\left(\frac{1}{3}\right)$
3. 0
4. 1
5. $\frac{4}{3}$
6. 3
7. $\frac{8}{3}$
8. 4
9. $h^{-1}(x)=\frac{x+6}{3}$
10. 2
11. 4
12. $[-3,13]$
13. D
14.
    a. $g(x)=2f\big(-(x-1)\big)$
       $a=2 \quad b=-1 \quad c=-1$
    b. $(-1,18)$
    c. $g(4)=-72$
15. B
16. D
17. C
18. A
19. C
20. A
21. C
22. B
23. $(-\infty,-3]\cup[5,\infty)$
24. $[-7,-3]$
25. $(-\infty,\infty)$
26. C
27. $-\frac{\pi}{4}$
28. $-\frac{\pi}{6}$
29. 0
30. $\frac{\pi}{6}$
31. 0
32. $\frac{3\pi}{4}$
33.
    a. 6
    b. 3
34. C
35. B
36. D
37. A, D, E
38. Period: 2 Amplitude: 12
39. A

40. $\left(-\frac{2\pi}{3}, 0\right) \cup \left(0, \frac{2\pi}{3}\right)$
41. $x = \sqrt{\frac{1}{2}}$ or $x = -\sqrt{\frac{1}{2}}$
42. $x = 2$
43.
    a. 3
    b. 4
    c. 3
44.
    a. $(f \circ g)(x) = \frac{1}{x-1}$ for $x \geq -3$
    b. B, C, D, E, G, H
45. Domain of $r^{-1}(x)$ is $[0,4]$ (since this is the range of $r(x)$)
    Range of $r^{-1}(x)$ is $[-3,2]$ (since this is the domain of $r(x)$)
46.



47.
    (A) $F\ (-40,85)$
    $G\ (5,45)$
    $J\ (50,5)$
    $K\ (95,45)$
    $P\ (140, 85)$
    (B) $h(t) = 40\cos\left(\frac{\pi}{90}(t+40)\right) + 45$
    Another correct answer: $h(t) = -40\cos\left(\frac{\pi}{90}(t-50)\right) + 45$
    Another correct answer: $h(t) = 40\cos\left(\frac{\pi}{90}(t-140)\right) + 45$
    (C) (i) h is positive and decreasing
    (ii) The graph of h is concave up on the interval (t_1, t_2). Thus, the rate of change of h is increasing on the interval (t_1, t_2).